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Effects of Blowing/Suction on Vortex Instability of Horizontal Free Convection Flows

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Introduction

THE problems of the vortex mode of instability in laminar free convection flow over a heated plate in a viscous fluid have been studied extensively. The instability mechanism is due to the presence of a buoyancy force component in the direction normal to the plate surface. The appearances of longitudinal vortices were observed by Sparrow and Husar.[1] Linear stability analyses were performed by Hwang and Cheng,[2] Haaland and Sparrow,[3] Kahawita and Meroney,[4] and Chen et al.[5–7] In these analyses, a quasiparallel flow model is assumed wherein the streamwise dependence of the basic flow is not neglected, but the disturbances are assumed to be independent of the streamwise direction. Recently, Lee et al.[8,9] applied a nonparallel flow model in which the streamwise dependence of the disturbance amplitude functions is taken into account, to re-examine the vortex instability of natural convection boundary-layer flows. The nonparallel flow analysis provides a larger critical Grashof number than the quasiparallel analysis, thus bringing the prediction closer to available experimental data.

All of the works mentioned above are only for flows over an impermeable surface. Free convection with blowing and suction over a vertical plate was studied.[10–12] Recently, Lee and Hsu[13] investigated the effects of blowing or suction on mixed convection over an inclined plate. However, the influence of blowing and suction on the flow and vortex instability of free convection boundary-layer flow over a horizontal surface does not seem to have been investigated. This has motivated the present investigation.

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Mathematical Formation

Base Flow

Consider a semi-infinite horizontal permeable surface with uniform temperature T_w and nonuniform surface mass flux v_w. The x coordinate represents the distance along the plate from its leading edge, and the y coordinate the distance normal to the surface. The surface mass flux is assumed to be a power function of x, i.e., $v_w = Ax^m$, where A and m are constants with $A > 0$ for blowing and $A < 0$ for suction. Under the assumption of constant fluid properties, along with application of the Boussinesq and boundary-layer approximations, the laminar governing equations can be written as[5]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta\frac{\partial}{\partial x}\int_y^\infty (T - T_\infty)\,\mathrm{d}y + \nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where u and v are the velocity components in the x and y directions; T is the temperature; β, α, and ν are the coefficients of thermal expansion, thermal diffusivity, and kinematic viscosity of the fluid; and g is the gravitational acceleration.

The boundary conditions for this problem are

$$\begin{aligned} &x = 0; && T = T_\infty,\ u = 0,\ v = 0 \\ &x > 0,\ y = 0; && T = T_w,\ u = 0,\ v = v_w = Ax^m \\ &y \to \infty; && T = T_\infty,\ u = 0 \end{aligned} \tag{4}$$

On introducing the following transformation:

$$\eta = \frac{y}{x}\left(\frac{Gr_x}{5}\right)^{1/5}, \quad f(\eta) = \frac{\psi(x, y)}{5\nu(Gr_x/5)^{1/5}}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{5}$$

where ψ is the stream function which automatically satisfies the continuity Eq. (1), and $Gr_x = [g\beta(T_w - T_\infty)x^3]/\nu^2$ is the local Grashof number. Equations (1–3) can be nondimensionalized as follows:[5]

$$f''' + 3ff'' - (f')^2 + \frac{2}{5}\left(\eta\theta + \int_\eta^\infty \theta\,\mathrm{d}\eta\right) = 0 \tag{6}$$

$$\frac{1}{Pr}\theta'' + 3f\theta' = 0 \tag{7}$$

with the boundary conditions

$$\theta(0) = 1,\ f'(0) = 0,\ f(0) = f_w = \frac{-Ax^{m+0.4}}{3\nu\left[\dfrac{g\beta(T_w - T_\infty)}{5\nu^2}\right]^{1/5}}$$

$$\theta(\infty) = 0,\ f'(\infty) = 0 \tag{8}$$

where the primes denote differentiation with respect to η and $Pr = \upsilon/\alpha$ is the fluid Prandtl number. It can be seen that similarity exists only for the case of $m = -0.4$. It is noted that $f_w = (-A/3\upsilon)[g\beta(T_w - T_\infty)/5\upsilon^2]^{-1/5}$ is the dimensionless blowing or suction parameter. f_w is positive for suction ($A < 0$), negative for blowing ($A > 0$), and $f_w = 0$ for the case of an impermeable surface.[8]

Disturbance Flow

Using standard methods of linear stability analysis with a nonparallel model,[8,9] one obtains the following system of dimensionless equations for the disturbance amplitude functions U, V, and Θ:

$$U'' + a_1U' + a_2U + a_3V = 0 \tag{9}$$

$$V'''' + b_1V''' + b_2V'' + b_3V' + b_4V + b_5U + b_6\Theta = 0 \tag{10}$$

$$\Theta'' + c_1\Theta' + c_2\Theta + c_3U + c_4V = 0 \tag{11}$$

along with boundary conditions

$$U(0) = V(0) = V'(0) = \Theta(0) = U(\infty) = V(\infty) = V'(\infty) = \Theta(\infty) = 0 \tag{12}$$

The coefficients in Eqs. (9–11) are defined in Ref. 8. Equations (9–12) constitute an eighth-order system of linear ordinary differential equations for the disturbance amplitude distributions $U(\eta)$, $V(\eta)$, and $\Theta(\eta)$. For fixed Pr and f_w, solutions of $U(\eta)$, $V(\eta)$ and $\Theta(\eta)$ are the eigenfunctions for the eigenvalues Gr_x and k, where $k = ax/(Gr_x/5)^{1/5}$ is the dimensionless spanwise wave number.

Results and Discussion

The system of Eqs. (6–8) for $m = -0.4$ was solved by a sixth-order variable step-size Runge-Kutta integration routine. The solutions for U, V, and Θ of the disturbance Eqs. (9–12) were obtained as the sum of four linearly independent solutions by numerical integration. The integration was performed from the outer edge of the boundary layer to the wall using the sixth-order Runge-Kutta variable step-size integration routine, along with the Gram-Schmidt orthogonalization procedure[14] to maintain the linear independence of the four individual solutions.

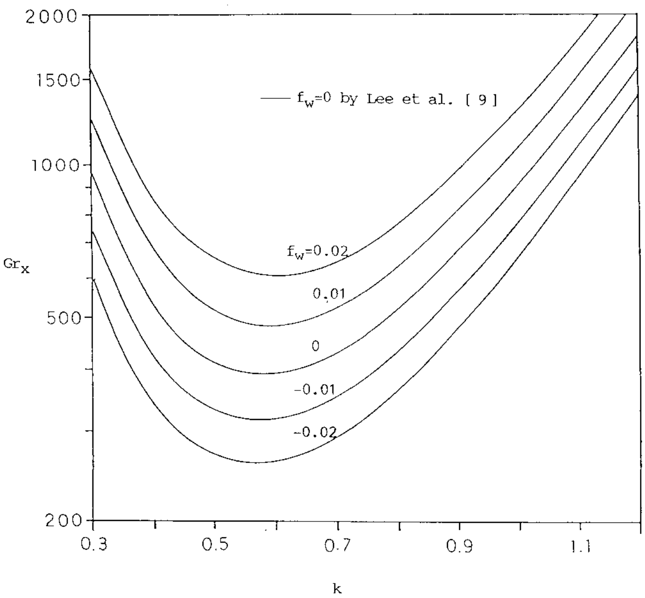

Fig. 1 Neutral stability curves for various values of f_w, $Pr = 0.7$ (air).

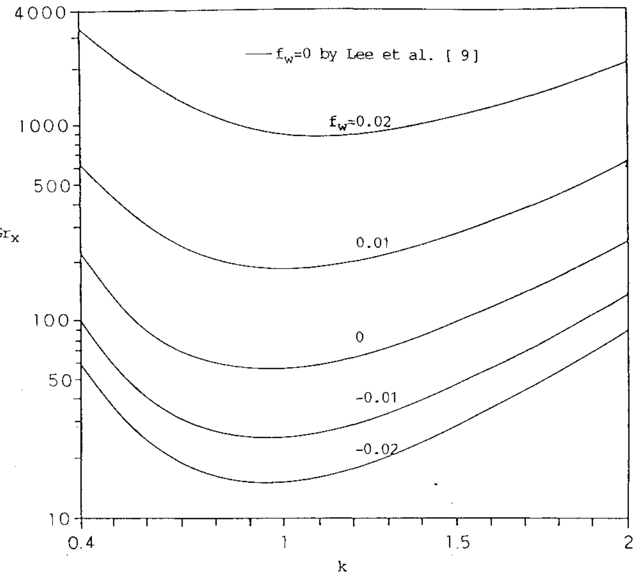

Fig. 2 Neutral stability curves for various values of f_w, $Pr = 7$ (water).

Table 1 Critical Grashof number (Gr_x^*) and the associated wave number (k^*) for various values of f_w, $Pr = 0.7$ and $Pr = 7$

	$Pr = 0.7$		$Pr = 7$	
f_w	Gr_x^*	k^*	Gr_x^*	k^*
−0.1	255.5	0.560	15.23	0.910
−0.09	266.5	0.562	16.56	0.915
−0.08	277.6	0.564	18.18	0.920
−0.07	289.4	0.567	20.20	0.925
−0.06	301.5	0.569	22.60	0.930
−0.05	313.4	0.571	25.65	0.935
−0.04	327.2	0.573	29.44	0.940
−0.03	341.2	0.576	34.22	0.945
−0.02	356.1	0.578	40.25	0.950
−0.01	372.6	0.580	47.92	0.955
0	388.5	0.582	57.94	0.960
0.01	403.6	0.584	71.05	0.965
0.02	421.9	0.587	88.27	0.970
0.03	439.7	0.589	111.5	0.975
0.04	459.9	0.591	143.7	0.980
0.05	480.5	0.593	187.7	0.985
0.06	503.6	0.596	247.7	0.990
0.07	526.2	0.598	332.8	0.995
0.08	548.0	0.600	455.6	1.000
0.09	572.6	0.602	636.7	1.005
0.1	600.6	0.604	883.5	1.100

Numerical results for the neutral stability curves and the critical Grashof number at the onset of vortex instability are presented for Prandtl numbers $Pr = 0.7$ (air) and 7 (water) over a wide range of the dimensionless blowing, and suction parameter f_w from −0.1 to 0.1. In order for the boundary-layer assumptions ($\upsilon << u$) to be valid in the present analysis, the values of f_w are limited in the range of −0.1 to 0.1.

Figures 1 and 2 show the neutral stability curves for air ($Pr = 0.7$) and water ($Pr = 7$), respectively, in terms of the Grashof number Gr_x and the dimensionless wave number k for selected values of f_w. It is observed that for suction ($f_w > 0$) as f_w increases (i.e., stronger suction), the neutral curves shift to higher Grashof numbers and higher wave numbers, indicating a stabilization of the flow to the vortex instability, while for blowing ($f_w < 0$) as $|f_w|$ increases (i.e., stronger blowing), the neutral curves shift to lower Grashof numbers and lower wave numbers, indicating a destabilization of the flow.

The critical Grashof number Gr_x^* and wave number k^*, which marks the onset of vortex instability, can be found from the minima of the neutral stability curves. These critical values for $Pr = 0.7$ and 7 are listed in Table 1. For $Pr = 7$ and $f_w = 0$, the critical Grashof number and wave number are computed to be 57.94 and 0.96, respectively, as compared to 56.326 and 0.942 reported in Lee et al.[8] appropriate for an impermeable surface. The results indicate that suction tends to stabilize the flow, while blowing tends to destabilize it. The reason that suction gives rise to a more stable flow is due to the fact that the effect of suction is to suck away the warm fluid on the plate and suppress the occurrence of vortices, and consequently suction stabilizes the vortex mode of instability. It is also seen that the critical wave number k^* increases as the flow changes from strong blowing to strong suction (i.e., f_w increases).

Conclusions

A dimensionless blowing or suction parameter $f_w = (-A/3v)[g\beta(T_w - T_\infty)/5v^2]^{-1/5}$ has been successfully employed in this note to investigate the effects of surface blowing and suction on the flow and vortex instability of a horizontal laminar natural convection boundary-layer flow. The numerical results demonstrate that blowing ($f_w < 0$) reduces the heat transfer rate and destabilizes the flow as compared with the case of an impermeable surface, while for suction ($f_w > 0$) the opposite trend is true.

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Mixed Convective Heating of a Moving Plate in a Parallel Duct

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Nomenclature

d = dimensionless thickness of moving plate, d^+/L_h
Gr = Grashof number, $g\beta(T_h - T_i)L^3/(\nu_f)^2$
g = gravitational acceleration
H_l = height of lower subchannel
H_u = height of upper subchannel
k = thermal conductivity
k^* = thermal conductivity ratio, k_p/k_f
L_{ds} = downstream duct length
L_{us} = upstream duct length
Nu = Nusselt number
Pe_p = plate Peclet number, u_pL_h/α_p
Pr = Prandtl number, ν_f/α_f
Re = Reynolds number, u_pL_h/ν_f
T = temperature
u_p = speed of moving plate
x, y = Cartesian coordinates
α = thermal diffusivity
β = thermal expansion coefficient
θ = dimensionless temperature, $(T - T_i)/(T_h - T_i)$
ν = kinematic viscosity
ϕ = inclination angle
Ψ = stream function
ω = vorticity

Subscripts
b = bottom plate surface or heater
f = fluid
h = heater
l = lower subchannel
p = moving plate
t = top plate surface or heater
u = upper subchannel

Introduction

VERY often heating or cooling of a moving surface can be encountered in various manufacturing processes such as the processes of rolling, extrusion, and continuous casting; drying and curing processes; and other thermal treatments of materials. After the pioneering investigations by Sakiadis[1] on the analysis of the boundary-layer flow induced on a continuous moving sheet or cylinder, heat and momentum transport phenomena due to a moving surface have been extensively studied for the past few decades. Comprehensive reviews on this subject can be found in recent works,[2–4] and therefore there is no need to repeat them here. It is noted that the previous works primarily focused on the boundary-layer cooling process of a moving surface in a quiescent ambient fluid of unconfined domain. In practice, however, the moving surface may be confined in a finite fluid space restricted by the neighboring solid boundaries. Moreover, the convection transport process induced by the moving surface within a confined domain can be expected to be more complex than that in an unconfined space. Therefore, an understanding of the convection heat transport phenomenon induced by a mov-

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